Discrete Fracture Network Modeling and Flow Simulations

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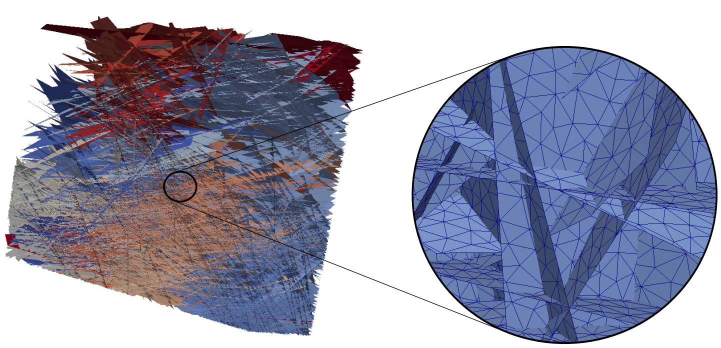
The DFN Module

Discrete fracture networks (DFN) are sets of mutually intersecting planar polygons resembling fractures in the underground.
DFN models are widely used to simulate underground phenomena and are particularly well suited, in contrast to homogenization methods, for transport phenomena, since fractures are individually represented, thus allowing for an accurate representation of flow paths.
The main difficulties to be addressed in dealing with simulations in DFNs are:

1. geometrical complexities: the generation of a mesh suitable for finite elements and conforming to interfaces (i.e. fracture intersections, or *traces*) on intricate networks of fractures often results infeasible or leads to poor quality elements;

2. domain size: networks for practical applications might count up to millions of fractures, each of them might have a fundamental impact on the flow properties, despite its size.



Enlargement of a mesh generation around intersections between fractures.

Darcy Flow

In a first approximation, the flow can occur only between the fractures without the possibility to spread in the rock matrix.
Taking into account some flow properties, the Darcy model has a very high reliability; its weak form and coupling equations for the DFNs, can be expressed as it follows:

$$
\begin{aligned}
&\int_{F_i} \mathbb{K}_i \nabla H_i \nabla v \,\mathrm{d}\Omega = \int_{F_i} q_i v \,\mathrm{d}\Omega + \int_{\Gamma_{iN}} G_i^N v_{|\Gamma_{iN}} \,\mathrm{d}\gamma \\
&\qquad + \sum_{S_m \in \mathcal{S}_i} \int_{S_m} \left[\!\left[\frac{\partial H_i}{\partial \hat{\nu}^i_{S_m}} \right]\!\right] v_{|S_m} \,\mathrm{d}\gamma, \qquad \forall v \in V_i,\ \forall i = 1, \dots, I, \\
&H_{i|S_m} - H_{j|S_m} = 0, \qquad \text{for } i, j \in I_{S_m},\ \forall m = 1, \dots, M, \\
&\left[\!\left[\frac{\partial H_i}{\partial \hat{\nu}^i_{S_m}} \right]\!\right] + \left[\!\left[\frac{\partial H_j}{\partial \hat{\nu}^j_{S_m}} \right]\!\right] = 0, \qquad \text{for } i, j \in I_{S_m},\ \forall m = 1, \dots, M,
\end{aligned}
$$

where $\Omega = \cup_{i=1,\dots,I} F_i$ denotes the DFN in the 3D space. $\mathbb{K}_i$ is a symmetric positive definite transmissivity tensor on F_i, while H denotes the global hydraulic head in the full DFN, and H_i its restriction to fracture F_i.

Darcy Flow: Optimization Approach

The previous problem can be reformulated as a PDE constrained optimization. In order to have a coercive constraint equation on each fracture F_i, a control variable $U_i^m \in \mathrm{H}^{-\frac{1}{2}}(S_m)$ for the constrained minimization problem is defined on each trace S_m as

$$
U_i^m = \left[\!\left[\frac{\partial H_i}{\partial \hat{\nu}^i_{S_m}} \right]\!\right] + \alpha H_{i|S_m},
$$

with a strictly positive parameter α.
So, the problem can be reformulated as:

$$
\begin{aligned}
\min J(H, U) \quad := \sum_{m=1}^{M} \Bigg(& \left\| H_{i|S_m} - H_{j|S_m} \right\|^2_{\mathrm{H}^{\frac{1}{2}}(S_m)} \\
+ \quad & \left\| U_i^m + U_j^m - \alpha \left(H_{i|S_m} + H_{j|S_m} \right) \right\|^2_{\mathrm{H}^{-\frac{1}{2}}(S_m)} \Bigg),
\end{aligned}
$$

with $i, j \in I_m$ and such that

$$
\begin{aligned}
&\int_{F_i} \mathbb{K}_i \nabla H_i \nabla v \,\mathrm{d}\Omega + \alpha \sum_{S_m \in \mathcal{S}_i} \int_{S_m} H_{i|S_m} v_{|S_m} \,\mathrm{d}\gamma = \int_{F_i} q_i v \,\mathrm{d}\Omega + \int_{\Gamma_{iN}} G_i^N v_{|\Gamma_{iN}} \,\mathrm{d}\gamma \\
&\qquad + \sum_{S_m \in \mathcal{S}_i} \int_{S_m} U_i^m v_{|S_m} \,\mathrm{d}\gamma, \qquad \forall v \in V_i,\ \forall i = 1, \dots, I.
\end{aligned}
$$

Conclusions

Generating a conforming mesh is a very complex task for DFNs simulations; in fact, we have to consider that, in a single fracture, hundreds of traces could be presented, each of which might have a fundamental impact on the flow properties, despite its size, making their management very hard to handle.

It has been proposed a new method, which overcome the complexities formerly stated, based on an optimization approach that relax the constrain of the conformity of the mesh, and manage each fracture singularly as a domain decomposition, which can be massively parallelized.

We want to exploit all this parallelism, both starting to use an hybrid approach (MPI+OpenMP and/or MPI+CUDA) and optimizing our MPI pure framework and its CUDA porting.

Bibliography

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